Activity 12 Scalar product (dot product)

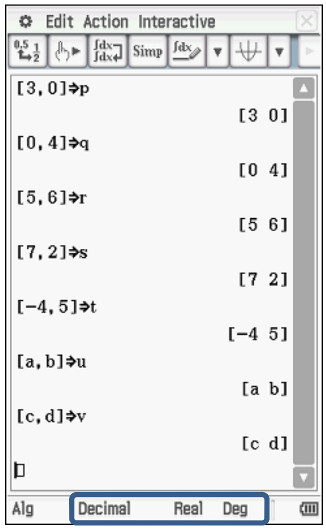
Aim: Develop the concept of scalar product and use it to solve closest approach problems.

Vectors **p** to **v** are defined as

p = [3, 0] **q** = [0, 4] **r** = [5, 6] **s** = [7, 2] **t** = [–4, 5] **u** = [a, b] **v** = [c, d]

Store the vectors p to v in Main

- Open Main
- Select [Edit | Clear All Variables]
- Ensure ClassPad is in Decimal and Degree modes
- Enter and store the vectors as shown



1. Complete the calculations (select [Action | Vector | dotP]).

a) dotP(p,q)

b) dotP(p,r)

c) dotP(q,r)

d) dotP(r,s)

e) dotP(s,t)

f) dotP(s,u)

g) dotP(u,v)

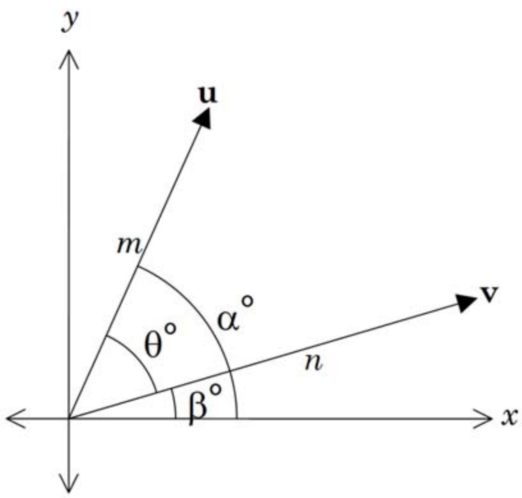
h) dotP(u,[2b,–2a])

2.

a) **p** and **q** are perpendicular vectors. What do your calculations in question 1 suggest about the scalar (dot) product of perpendicular vectors?

b) For vectors **u** = [a, b] and **v** = [c, d], the scalar product is written as **u•v** (pronounced "u dot v"). Write a formula in terms of a, b, c and d for **u•v**.

3. An alternative magnitude and direction formula for the scalar product can be found as follows.



a) Write vectors **u** and **v** in polar form.

Clear all variables before proceeding.

b) Store vectors **u** and **v**, in polar form in your ClassPad. What is the output?

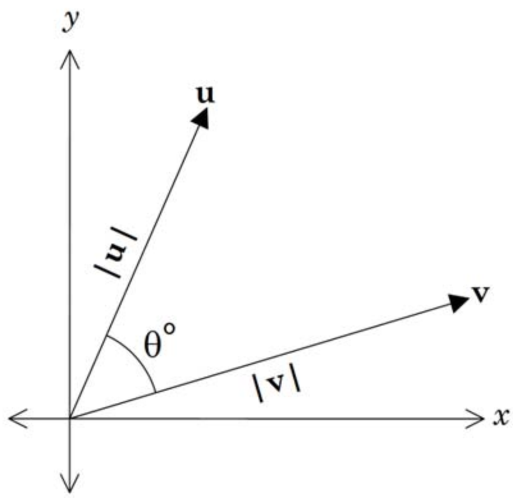
c) Determine dotP(u,v).

d) Enter tCollect(ans) and record the output.

e) Hence write a formula for the scalar product of two vectors given in magnitude and direction form.

4. To prove $|\mathbf{u}||\mathbf{v}|\cos\theta = ac + bd$

a) Apply the cosine rule to the adjacent triangle to write an expression for $|\mathbf{v} - \mathbf{u}|^2$



b) Substitute for $\mathbf{u} = [a,\ b]$ and $\mathbf{v} = [c,\ d]$ and rearrange to complete the proof.

The most significant property of the scalar product is that perpendicular vectors have a scalar product of zero.

Closest approach

5. An illegal fishing vessel is sighted at 10 am at position vector [4, –26] km travelling with constant velocity [4, 12] km/h. A patrol boat is moored at position vector [17, 3] km.

   a) Write down the position vector $\mathbf{r}_f(t)$ of the fishing boat t hours after 10 am.

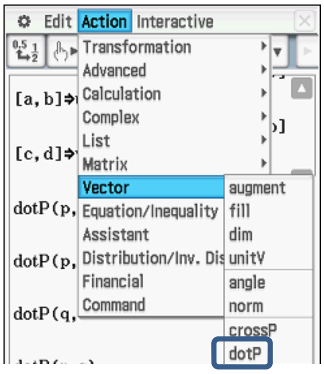
   b) Determine the separation vector between the patrol boat and the fishing vessel, i.e. the relative vector ${}_f\mathbf{r}_p(t)$.

   c) Use the scalar product to help you determine the time at which the fishing vessel is closest to the patrol boat, its position at this time and the minimum distance between the two boats.

Learning notes

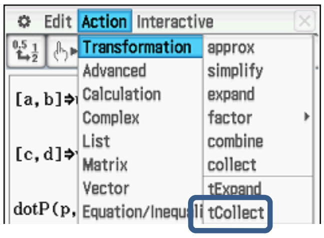
Calculate dot product

- Select [Action | Vector | dotP]



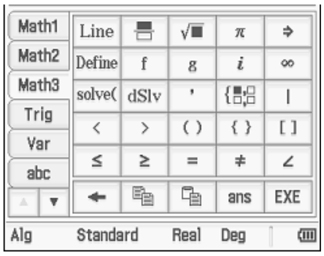
The tCollect command (simplification of trigonometric expressions)

- Select [Action | Transformation | tCollect].



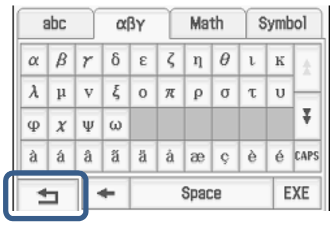
Enter vectors in magnitude and direction form

- Use the [Keyboard] [Math3] tab to use the angle symbol ∠, for example, vector **u** in Q3 can be stored using [m,∠(α)]⇒u.



Greek letters can be found in the alphabet keyboard

- Tap [abc] to open the alphabet keyboard(s)
- Tap the [αβγ] tab for the Greek letters
- Tap [⮌] to go back to the regular keyboard



Q5 The boats are closest when the separation vector and the relative velocity vector are perpendicular, i.e the dot product is 0.

Why do perpendicular vectors have dot product of 0?

In the diagram triangle ABF is rotated 90° about point B, i.e. AB and BC are perpendicular. Note:

$\overrightarrow{BA} = [d, e]$

$\overrightarrow{BC} = [e, -d]$

$\overrightarrow{BA} \bullet \overrightarrow{BC} = de - ed = 0$

Scalar multiples of $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are also perpendicular and have dot product of 0.